Unsteady flow around tandem spheres at $Re_D = 3900$

1 General description

The aim of this test case is to asses the accuracy and efficiency of high-order solvers for the prediction of complex unsteady multi-scale flow under low Mach and low Reynolds number conditions. A uniform free stream initial condition is specified from the Mach number ($M_\infty = 0.1$), Reynolds number based on single sphere diameter, D, ($Re_D = 3900$), and Prandlt number ($Pr = 0.72$). Solution time is non-dimensionalized by the characteristic time scale:

$$t^* = t\frac{U_\infty}{D},$$

where t^* is the non-dimensional time, t is the dimensional time, and U_∞ is the free stream velocity. To enable a manageable work load through spatial and temporal resolution studies comparisons of quantities of interest are to be carried out on the non-dimensional time interval $t^* \in [100, 200]$.

Performing an assessment of the order of accuracy of solvers is a prerequisite for this case. Complete the Taylor Green vortex case (`https://how5.cenaero.be/content/ws1-dns-taylor-green-vortex-re1600`) to asses the to asses the grid convergence of the kinetic energy in the domain, E_k, at $t/tc = 8$.

2 Governing Equations

The governing equations are the full 3D compressible Navier-Stokes equations with a constant ratio of specific heats ($\gamma = 1.4$), a constant Prandtl number ($Pr = 0.72$). Sutherland's law is used to model temperature dependent viscosity. The initial conditions and inflow are specified at Mach ($M_\infty = 0.1$) as $\vec{u} = \{U_\infty, 0, 0\}$. Please employ implicit LES (no wall models, or sub-grid models), A.K.A. an unsteady-laminar simulation with the discretization providing filtering.

3 Geometry

Two spheres of diameter D whose centers are separated by $10D$ along the stream wise centerline of the domain. The center of the up stream sphere is located $\{0, 0, 0\}$. The center of the down stream sphere is located at $\{10D, 0, 0\}$. For clarity, differentiate results from each sphere with the labels up stream and down stream.

4 Flow conditions

The free stream Mach number is ($M_\infty = 0.1$), the Reynolds number based on sphere diameter D is ($Re_D = 3900$), and the angle of attack is $0°$. The free stream temperature is $T_\infty = 300\,K$, and density $\rho_\infty = 1.225\,kg/m^3$.

5 Boundary Conditions

- Far field: characteristic
- Sphere surfaces: adiabatic wall

6 Mandatory computations and results

Run the Taylor Green vortex case (`https://how5.cenaero.be/content/ws1-dns-taylor-green-vortex-re1600`). Conduct a grid independence on Cartesian and unstructured/perturbed as described in the full test case description (`https://how5.cenaero.be/sites/how5.cenaero.be/files/BS1_TaylorGreenVortexRe1600_0.pdf`). Plot the order of accuracy for the kinetic energy in the domain, E_k, at $t/tc = 8$ for each grid resolution (64^3, 128^3, 256^3).

Mesh families for order $P1$ through $P4$ are provided by Steve Karman of Pointwise, Inc. Conduct a spatial and temporal resolution study for as many orders as possibly to determine the combinations that produce a converged average drag coefficient (C_D) and Strouhal Number (St) based on lift for the down stream sphere during

$t^* \in [100, 200]$.

Using the mesh(es) and time step(s) that produce converged C_D and St for the down stream sphere report the following quantities from measurements made during $t^* \in [100, 200]$:

1. Integral quantities for both spheres:
   (a) Mean values: lift coefficient C_L, drag coefficient C_D
   (b) Root-means-squared values: C_L, C_D
2. Surface quantities on $x - y$, $x - z$, and $y - z$ planes passing through the center of each sphere:
   (a) Mean values: pressure coefficient C_P, skin friction coefficient C_f
   (b) Root-means-squared values: C_P, C_f
3. Flow Samples
   (a) Quantities
      i. Mean values on stream wise and transverse transects:
         A. u, v, and w velocity components (non-dimensionalized by U_∞)
         B. Reynolds stresses $(u'u', u'v', v'v')$ (non-dimensionalized by U_∞^2)
      ii. Root-means-squared values on stream wise and transverse transects:
         A. u, v, and w velocity components (non-dimensionalized by U_∞)
         B. Reynolds stresses $(u'u', u'v', v'v')$ (non-dimensionalized by U_∞^2)
      iii. Frequency spectra at points:
         A. total velocity
         B. pressure coefficient C_P
         C. turbulent kinetic energy
   (b) Locations
      i. Stream wise 1: $[\{-5.5D,\ 0,\ 0\},\ \{-0.5D,\ 0,\ 0\}]$ from 5.5 diameters up stream to the leading edge of the up stream sphere
      ii. Stream wise 2: $[\{0.5D,\ 0,\ 0\},\ \{9.5D,\ 0,\ 0\}]$ from the trailing edge of the up stream sphere to the leading edge of the down stream sphere
      iii. Stream wise 3: $[\{10.5D,\ 0,\ 0\},\ \{15.5D,\ 0,\ 0\}]$ from the trailing edge of the down stream sphere to 15.5 diameters down stream

iv. y Transverse 1: $[\{-1.5D,\, -D,\, 0\},\, \{-1.5D,\, D,\, 0\}]$ along a two diameter span located 1.5 diameters up stream of the up stream sphere

v. y Transverse 2: $[\{1.5D,\, -D,\, 0\},\, \{1.5D,\, D,\, 0\}]$ along a two diameter span located 1.5 diameters down stream of the up stream sphere

vi. y Transverse 3: $[\{5D,\, -D,\, 0\},\, \{5D,\, D,\, 0\}]$ along a two diameter span located mid-way between the spheres

vii. y Transverse 4: $[\{8.5D,\, -D,\, 0\},\, \{8.5D,\, D,\, 0\}]$ along a two diameter span located 1.5 diameters up stream of the down stream sphere

viii. y Transverse 5: $[\{11.5D,\, -D,\, 0\},\, \{11.5D,\, D,\, 0\}]$ along a two diameter span located 1.5 diameters down stream of the down stream sphere

ix. y Transverse 6: $[\{15D,\, -D,\, 0\},\, \{15D,\, D,\, 0\}]$ along a two diameter span located 5 diameters down stream of the down stream sphere

x. z Transverse 1: $[\{-1.5D,\, 0, D\},\, \{-1.5D,\, 0,\, D\}]$ along a two diameter span located 1.5 diameters up stream of the up stream sphere

xi. z Transverse 2: $[\{1.5D,\, 0, D\},\, \{1.5D,\, 0,\, D\}]$ along a two diameter span located 1.5 diameters down stream of the up stream sphere

xii. z Transverse 3: $[\{5D,\, 0, D\},\, \{5D,\, 0,\, D\}]$ along a two diameter span located mid-way between the spheres

xiii. z Transverse 4: $[\{8.5D,\, 0, D\},\, \{8.5D,\, 0,\, D\}]$ along a two diameter span located 1.5 diameters up stream of the down stream sphere

xiv. z Transverse 5: $[\{11.5D,\, 0, D\},\, \{11.5D,\, 0,\, D\}]$ along a two diameter span located 1.5 diameters down stream of the down stream sphere

xv. z Transverse 6: $[\{15D,\, 0, D\},\, \{15D,\, 0,\, D\}]$ along a two diameter span located 5 diameters down stream of the down stream sphere

xvi. Point 1: $\{-2.5D,\, D,\, D\}$

xvii. Point 2: $\{5D,\, D,\, D\}$

xviii. Point 3: $\{12.5D,\, D,\, D\}$

4. Provide details of the computational resources utilized in terms of DOF, work units, and time marching scheme.

5. Provide details of the computational hardware and parallelization strategy utilized.

Report non-dimensional quantities. For example, scale Reynolds stresses by the square of the free stream velocity.

The visualizations of this case shown below were prepared by Kevin Holst from simulations he conducted. Figures 1 and 2 display snapshots of iso-surfaces of Q-criterion colored by vorticity magnitude and the magnitude of vorticity on the $x - y$ plane, respectively. The videos in Figures 3 and 4 display iso-surfaces of Q-criterion colored by vorticity magnitude and the magnitude of vorticity on the $x - y$ plane during $t^* \in [0, 100]$, respectively. The simulation that produced the data displayed in the figures and videos was run with a non-dimensional time step $\Delta t^* = 0.003$ on the fourth $P2$ mesh with approximately 9 million DOF. The videos plays back every 500^{th} time step at 10 frames per second.

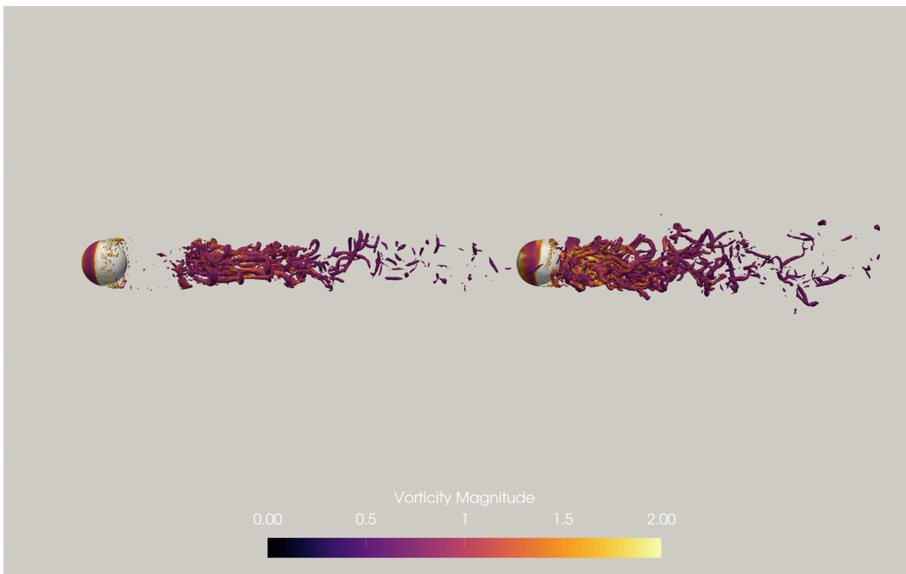

Figure 1: Visualization of the near wake region with iso-surfaces of Q-criterion colored by vorticity magnitude.

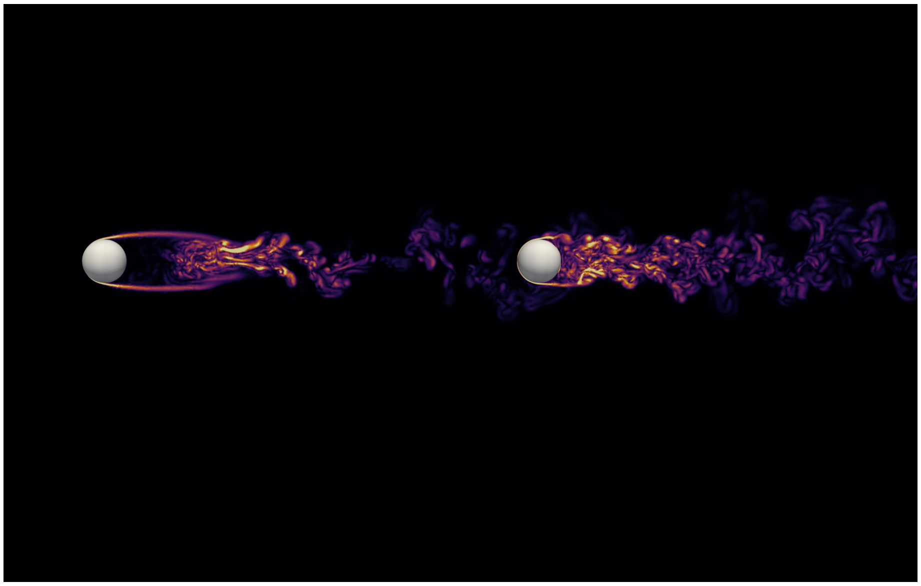

Figure 2: Visualization of the near wake region with vorticity magnitude on a plane passing through the sphere centers.

Figure 3: Video displaying iso-surfaces of Q-criterion colored by vorticity magnitude at 10 frames per second.

Figure 4: Video displaying vorticity magnitude on the $x - y$ plane at 10 frames per second.