

# VI2 - Inviscid Flow through a Channel with a Smooth Gaussian Bump

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## General description

This problem is aimed at testing high-order methods for the computation of internal flow with a high-order curved boundary representation. In this subsonic flow problem, the geometry is smooth, and so is the flow. Entropy should be a constant in the flow field. The  $L_2$  norm of the entropy error is then used as the indicator of solution accuracy since the analytical solution is unknown. The convergence rate can be expected to be  $P+1$ , where  $P$  is order of the discrete polynomial approximation.

## Governing Equations and models

The governing equation is the 2D Euler equations with a constant ratio of specific heats of  $\gamma = 1.4$ .

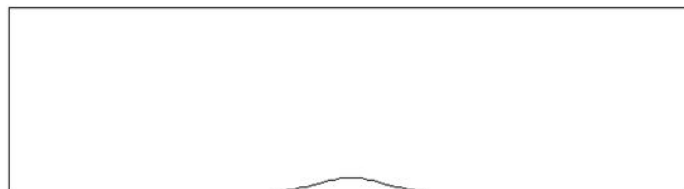
## Boundary Conditions

The inflow Mach number is  $M = 0.5$  with 0 degree angle of attack. Total temperature,  $\frac{T_0}{T} = 1 + \frac{\gamma-1}{2}M^2$ , and total pressure,  $\frac{P_0}{P} = \left(\frac{T_0}{T}\right)^{\frac{\gamma}{\gamma-1}}$ , are imposed as an inflow boundary condition on the left boundary, and a constant static pressure is imposed on the outflow boundary. The upper and lower walls are slip walls.

## Geometry and grids

The computational domain is bounded between  $x = -1.5$  and  $x = 1.5$ , and between the bump and  $y = 0.8$ , as shown below. The bump is defined as

$$y = 0.0625e^{-25x^2}$$



The set of structured quad or triangle grids are generated by the python scripts provided on the web site. The type of meshes, the interpolation order and the number of refinements are specified through options specified at the end of the Smoothbump.py script.

## Mandatory campaign

The main objective is to demonstrate grid convergence of drag on a sequence of successively refined meshes. The provided structured meshes must be used for all calculations, but additional computations on unstructured meshes can be provided as well. A Python script to generate the meshes is provided to participants to generate the meshes in their preferred file format if needed.

1. Start the simulation from a uniform free stream with  $M = 0.5$  everywhere, and monitor the  $L_2$  norm of the density residual. Compute the work units required to achieve a steady state. Use the following non-dimensional entropy error as the accuracy indicator

$$Err_{L_2(\Omega)} = \sqrt{\frac{\int_{\Omega} \left( \frac{p}{p_{\infty}} \cdot \left( \frac{\rho_{\infty}}{\rho} \right)^{\gamma} - 1 \right)^2 dV}{V}}$$

2. Perform this exercise for at least three different meshes and with different orders of accuracy to assess the performance of high-order schemes of various accuracy.
3. Plot the  $L_2$  entropy error vs. work units to evaluate efficiency, and  $L_2$  entropy error vs. length scale  $h$  to assess the numerical order of accuracy.
4. The raw data should be provided in three columns,  $h = 1 / \sqrt{nDOFs}$ , Error, and work units. The data should be separated by different  $p$  values. An example format is provided below:

P = 0

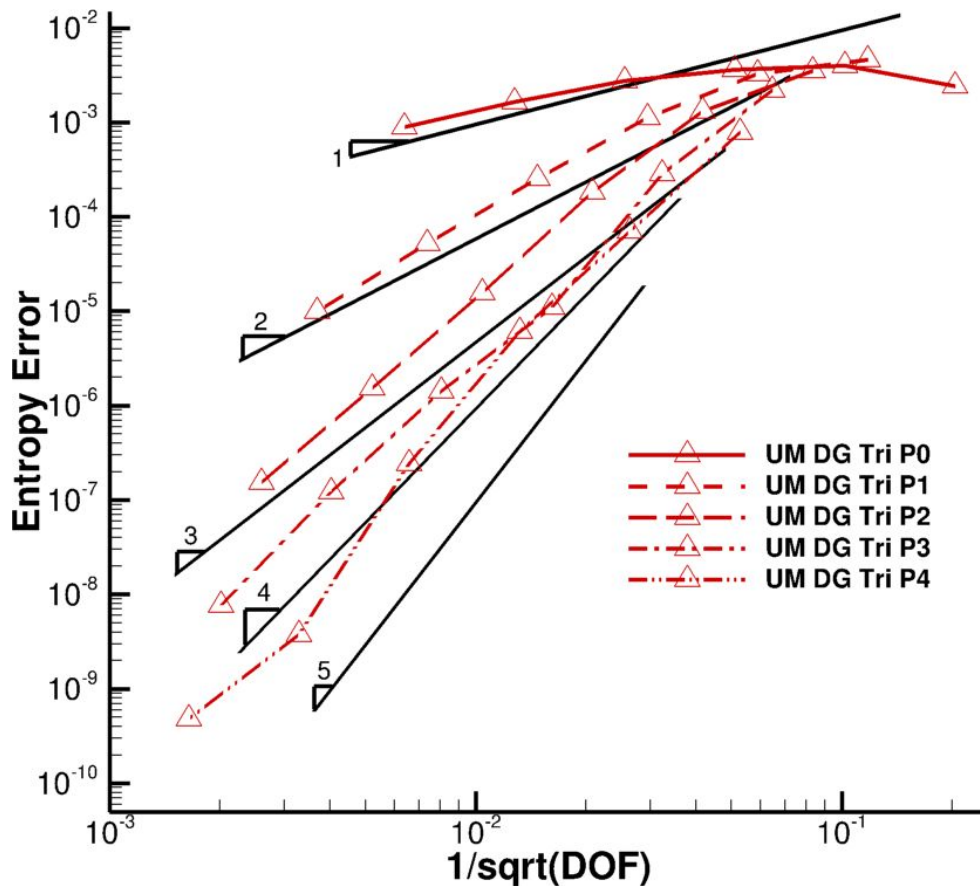
1/sqrt(DOF)	Entropy Error	Work Units (optional)
xxx	xxx	xxx
xxx	xxx	xxx

P = 1

1/sqrt(DOF)	Entropy Error	Work Units (optional)
xxx	xxx	xxx
xxx	xxx	xxx

## Presentation Guideline

Participants verification slide should include a plot of the order of accuracy that includes the reference slope lines as illustrated in the figure below. These slope lines provide a consistent reference across all presentations. The data for the slopes are given below, and provided in a tecplot format on the website.



Example order of accuracy plot that includes reference slope lines

## Reference slope lines

P=0 Slope of 1

1/sqrt(DOF)	Entropy Error
0.1443374406	0.0136643822
0.0045105552	0.0004270119

P=1 Slope of 2

1/sqrt(DOF)	Entropy Error
0.0721688857	0.0030347995
0.0022552776	2.96367106462231E-06

P=2 Slope of 3

1/sqrt(DOF)	Entropy Error
0.04811273	0.0005176307
0.0015035227	1.57968320206962E-08

P=3 Slope of 4

1/sqrt(DOF)	Entropy Error
0.0360844425	0.0001550279
0.0022552776	2.36553838701671E-09

P=4 Slope of 5

1/sqrt(DOF)	Entropy Error
0.0288675549	1.86685247549256E-05
0.0036084443	5.69718077365507E-10