Laminar Joukowski Airfoil

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General description

This test case is designed as a verification case of the viscous terms of the Navier-Stokes equations. Participants are required to use the provided grids, as they have been demonstrated to be able to provide the optimal convergence rate in drag coefficient. A low Reynolds number of 1000 is employed to emphasize the viscous terms. For an adjoint consistent discretization, the optimal convergence rate for an output functional is 2P. Otherwise, the convergence rate can be expected to be P+1. The Joukowski airfoil is used for this test as the cusped trailing edge removes the inviscid singularity at the trailing edge at zero degrees angle of attack. However, there is still a singularity in skin friction. The provided grids are design to cluster nodes at both the trailing edge singularity and the stagnation point in order to capture the expected order of accuracy. Hence, all participants must use the provided grids. Please do not hesitate to contact Marshall Galbraith as soon as possible if you are having difficulties using the provided grids.

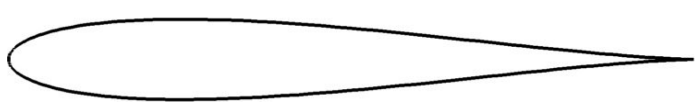
Governing Equations and models

The compressible Navier-Stokes equations should be used, with air as working medium. The freestream Mach number is 0.5, a Reynolds number of 1000 based on chord, an angle of attack of 0 degrees, and the heat capacity ratio is $\gamma=C_p/C_v=1.4$. The dynamic viscosity is constant. The Prandtl number is fixed to Pr=0.72.

Boundary Conditions

The far field boundary can be imposed with a Riemann invariant or characteristic boundary condition. Note that the far field boundary is not far enough away to be non-influential, in particular for high-order calculations. Hence the boundary condition used must be documented, and will likely lead to slight difference in the “truth” drag coefficient on the finest mesh. The airfoil surface is imposed as a no-slip adiabatic wall.

Geometry and grids



The geometry of the Joukowski airfoil is shown in the adjacent figure. Structured, unstructured quadrilateral, and unstructured triangular grids are provided via a set of Python scripts. The provided grids must be used by all participants, through custom and adapted grids are also welcomed in addition to the provided grids. A python script (Laminar.py) is provided that generates the grids. This allows participants to generate custom file formats if desired. Please contact Marshall Galbraith if you have any problems generating the grids.

Common Inconsistencies

The following is a list of common inconsistencies that can lead to computing a different "truth" drag coefficient value.

1. Using a different Prandtl number than 0.72.
2. Using Sutherland's law rather than constant viscosity.
3. Using isothermal wall rather than adiabatic wall.

Mandatory campaign

The main objective is to demonstrate grid convergence of drag coefficient on a sequence of successively refined meshes. The provided meshes must be used for all calculations, whereas additional computations on unstructured meshes will be accepted as well. Participants should provide a non-dimensional drag coefficient error for each of the grids. Due to variations in boundary condition implementations, participants should compute a reference drag coefficient on the finest grid with the highest order of accuracy available in their software. The drag coefficient error for each grid is computed relative to this reference drag coefficient. Participants should also verify that machine zero lift is computed on all grids.

1. Start the simulation from a uniform free stream with M = 0.5 everywhere, and monitor the L_2 norm of the density residual. Compute the work units required to achieve a steady state where the density residual has dropped at least 10 orders of magnitude.
2. Perform this exercise for at least three different meshes and with different orders of accuracy to assess the performance of high-order schemes of various accuracy.
2. Plot the drag coefficient error vs. work units to evaluate efficiency, and drag coefficient error vs. length scale to assess the numerical order of accuracy.
3. The raw data should be provided in three columns, $h = 1/\sqrt{nDOFs}$, drag coefficient, and work units. The data should be separated by different p values. An example format is provided below:

P = 0

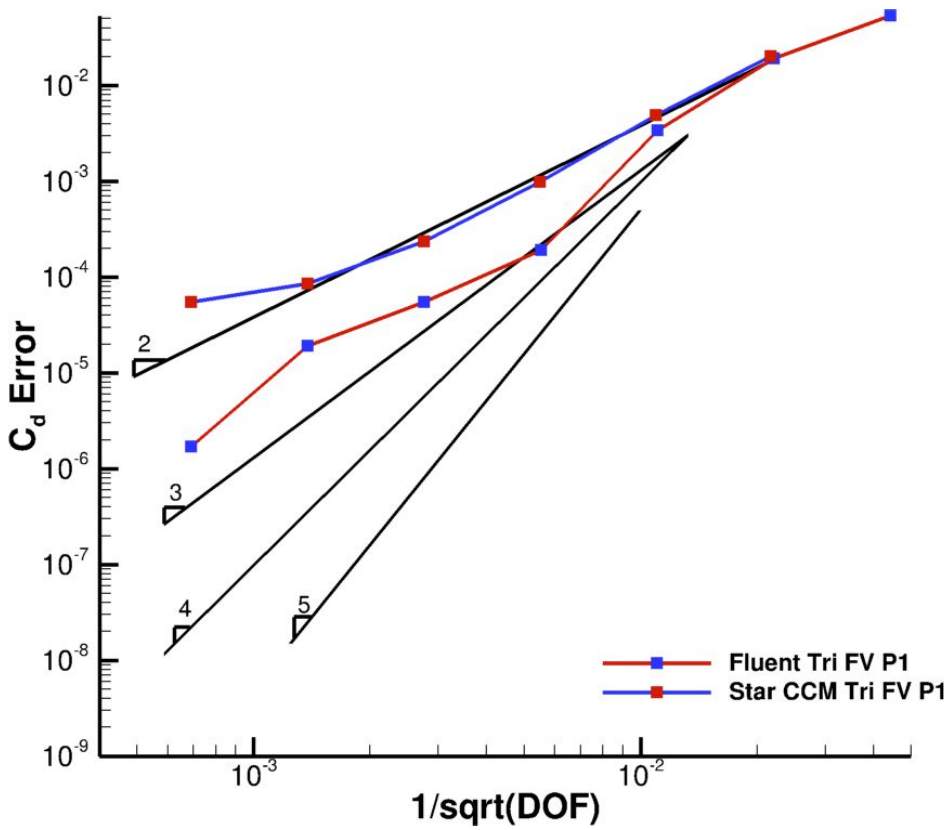
1/sqrt(DOF)	Cd	Work Units (optional)
xxx	xxx	xxx
xxx	xxx	xxx

P = 1

1/sqrt(DOF)	Cd	Work Units (optional)
xxx	xxx	xxx
xxx	xxx	xxx

Presentation Guideline

Participants verification slide should include a plot of the order of accuracy that includes the reference slope lines as illustrated in the figure below. These slope lines provide a consistent reference across all presentations. The data for the slopes are given below, and provided in a tecplot format on the website. Please use either the P+1 or 2P slopes depending on the expected order of accuracy of the scheme.



Example order of accuracy plot that includes P+1 reference slope lines

Reference slope lines for P+1 slopes

P=1 Slope of 2

1/sqrt(DOF)	Cd Error
0.0220970869	1.86E-002
0.0004882813	9.10224687351475E-06

P=2 Slope of 3

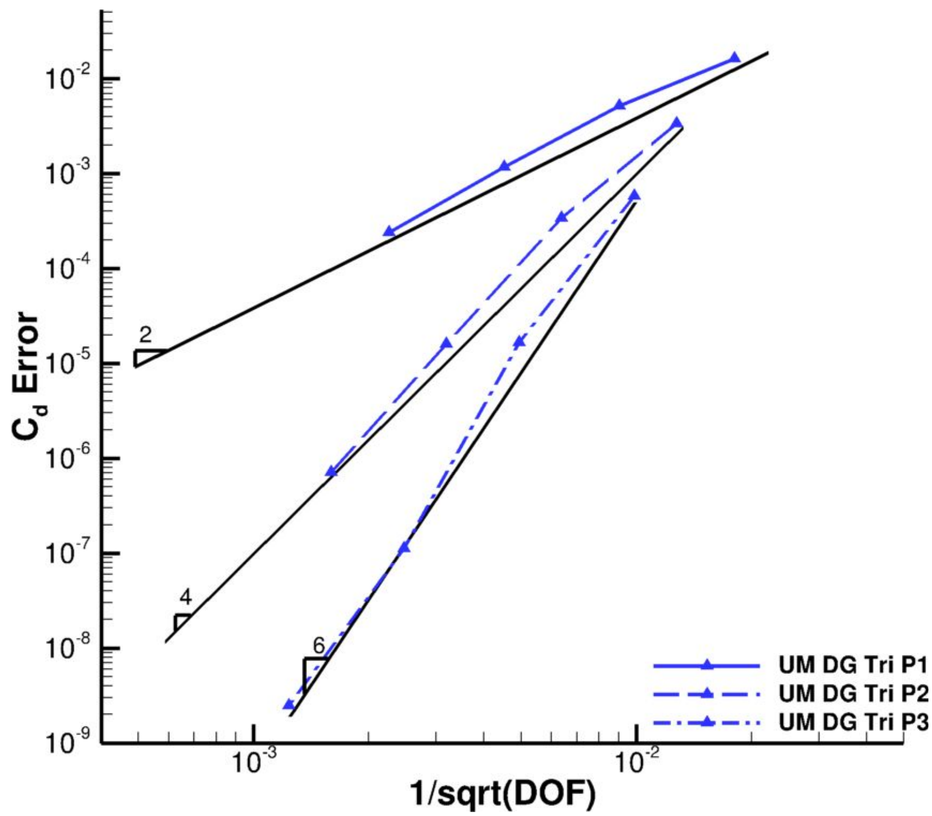
1/sqrt(DOF)	Cd Error
0.0132582521	3.02E-003
0.0005859375	2.6056752059198E-07

P=3 Slope of 4

1/sqrt(DOF)	Cd Error
0.0132582521	3.02E-003
0.0005859375	1.15155662979722E-08

P=4 Slope of 5

1/sqrt(DOF)	Cd Error
0.0099436891	4.93E-004
0.0012429611	1.5E-08



Example order of accuracy plot that includes 2P reference slope lines

Reference slope lines for 2P slopes

P=1 Slope of 2

1/sqrt(DOF)	Cd Error
0.0220970869	1.86E-002
0.0004882813	9.10224687351475E-06

P=2 Slope of 4

1/sqrt(DOF)	Cd Error
0.0132582521	3.02E-003
0.0005859375	1.15155662979722E-08

P=3 Slope of 6

1/sqrt(DOF)	Cd Error
0.0099436891	4.93E-004
0.0012429611	1.87939679276122E-09

P=4 Slope of 8

1/sqrt(DOF)	Cd Error
0.0092807765	6.47E-005
0.0023201941	1.0E-09